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Geometric Analysis of Shark Teeth

Leslie B. Jones, Daniel R. Huber, and Rebecca J. Waggett

The dawn of the scientific age coincided with the realization that nature can be described through mathematics. As stated by Galileo (1623), the entire universe is "written in the language of mathematics." Since then, quantitative analysis of natural phenomena has been at the heart of scientific inquiry. Due to the inherent connection between natural phenomena and its description via mathematics, nature provides a tangible context for mathematics that can be used to establish relevance, activate prior knowledge, and invite students into mathematics instruction.

The use of nature as a context for mathematics is extremely evident with regard to biology and geometry. Biological structures vary greatly in their geometry, and geometric differences among organisms have functional consequences that determine which organisms are better able to acquire resources (e.g., food, habitat). In other words, the ability to gather resources (i.e., the ability to be ecologically successful) is based on adequate performance, and adequate performance is based on advantageous geometry (Huber et al., 2006).

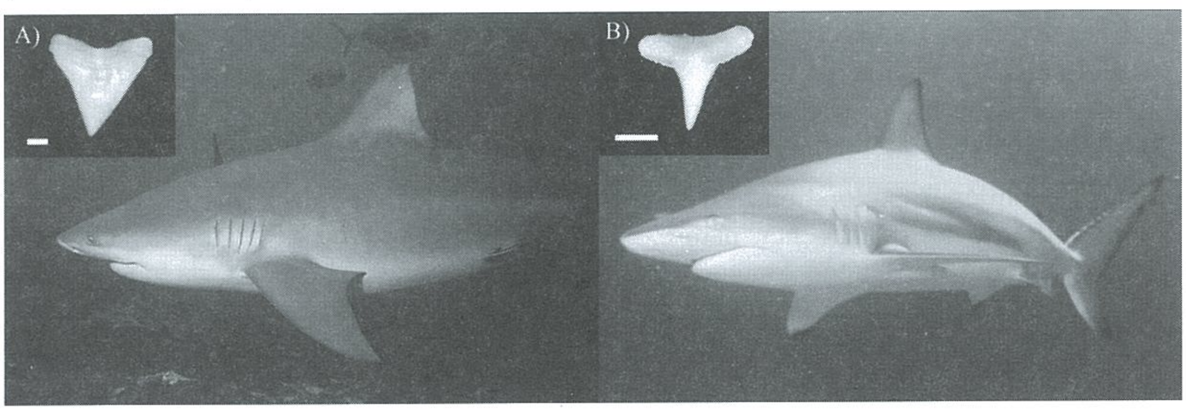

Figure 1 – A) The bull shark *Carcharhinus leucas* with inset of tooth from anterior region of upper jaw. B) The blacktip shark *Carcharhinus limbatus* with inset of tooth from anterior region of upper jaw. Tooth cusps are the more acute triangular areas below the horizontal tooth base. Scale bars indicate 5 mm.

One compelling example for exploring the impact of geometry on the biological success of a population is shark predation. Shark teeth vary greatly in their geometry, and this variability in form determines their cutting performance. Variability in cutting performance affects their feeding ecology, and causality can therefore be established between the geometric variability of the teeth and the feeding ecology of the shark (Whitenack & Motta 2010). A shark bite generally involves two behaviors: puncture and lateral head-shaking. Once the teeth have punctured into a prey item, the head is whipped from side to side (lateral head-shaking), sawing the teeth through the prey. The ability of a shark tooth to puncture a prey item is a function of the pressure generated along the surface of the tooth. However, the ability of a shark tooth to saw through prey during lateral head-shaking is a function of its width; wider teeth can better resist the stresses that are generated when the teeth contact the prey (Wainwright et al., 1976).

In this exercise, we consider two sharks, the bull shark *Carcharhinus leucas* and the blacktip shark *Carcharhinus limbatus* (Fig. 1). Both species capture small prey whole, but as the size of the prey increases relative to the size of their mouths, the sharks also employ lateral head-shaking to saw out chunks of flesh. The blacktip shark primarily consumes small to medium-sized fish, while the bull shark consumes small to large-sized fish, other sharks, and even sea turtles and mammals.

We will begin our analysis with consideration of tooth pressure. Tooth pressure is equal to bite force divided by the lateral surface area of the tooth in contact with the object being bitten:

$$Tooth\ Pressure = \frac{Bite\ Force}{Tooth\ Lateral\ Surface\ Area}$$

The bull shark tooth below is from a 120 cm shark, while the blacktip shark tooth is from a 152 cm shark (Fig. 2). At these respective lengths, we can assume that the bite force of these two sharks is the same (Huber et al. 2006, Habegger et al., 2012). We consider each tooth at $\frac{1}{3}$ the distance from the tip, accounting for the puncture depth of the bite. However, to extend the lesson or facilitate groups, other ratios may be used.

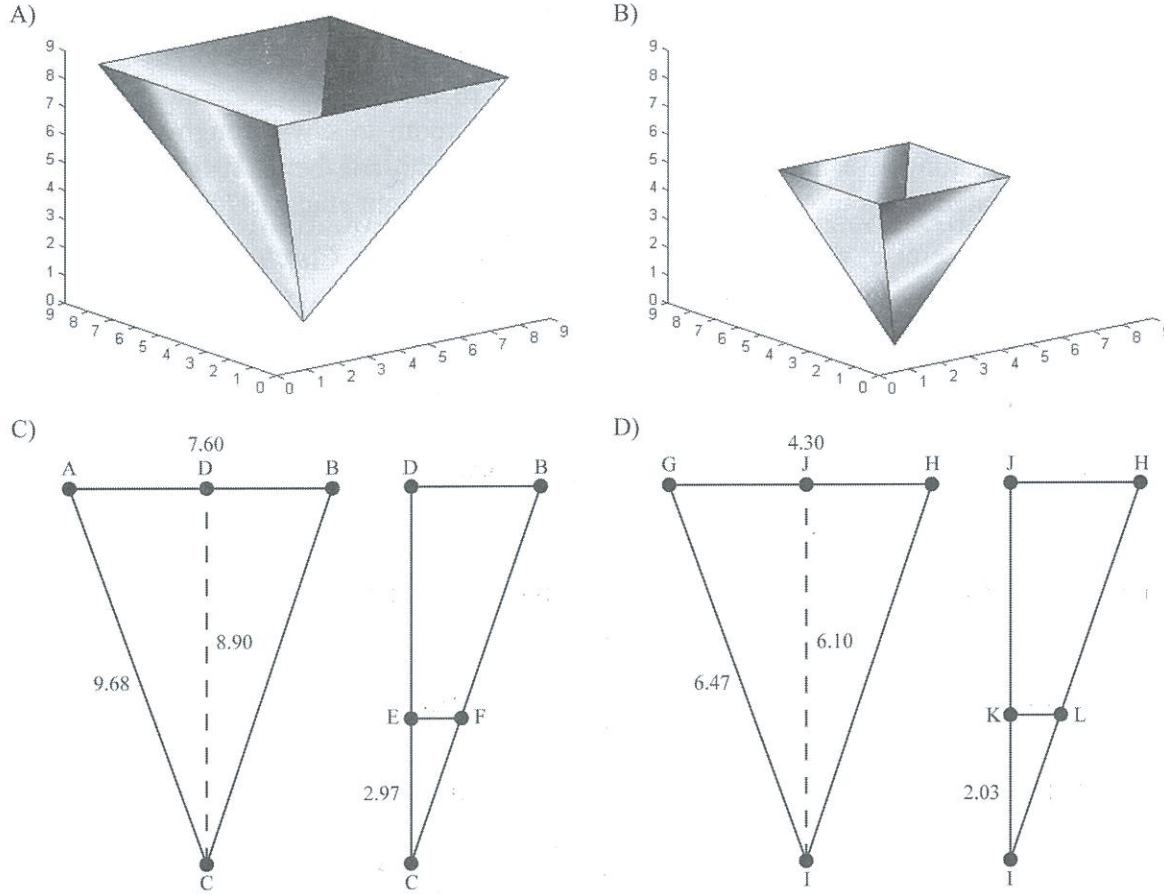

Figure 2 – 3-D representations of bull shark (A) and blacktip shark (B) teeth modeled as right-square pyramids. 2-D cross-sectional representations of bull shark (C) and blacktip shark (D) teeth modeled as right-square pyramids. In C and D the right images are teeth that have been vertically bisected and contain lines parallel to the base at one-third the altitude from the tip of each tooth. Images in A and B were generated with Matlab mathematical software while those in C and D were generated with GeoGebra freeware. Triangles are not drawn to scale. Tooth dimensions are based on measurements taken from bull and blacktip shark specimens at the Florida Museum of Natural History at the University of Florida. All measurements are in millimeters.

Shark Bite Part 1

1. If the bull shark and the blacktip shark have the same bite force, will the tooth with the larger or smaller lateral surface area in contact with the prey item have the greater tooth pressure?

2. Compute the lateral surface area of the bull shark tooth.

3. Compute the lateral surface area of the blacktip shark tooth.

4. In the first model of figure 2(C), which shows triangle CAB, $\overline{CD}$ is a perpendicular bisector of $\overline{AB}$. In the second model, half of triangle CAB is shown.

   In triangle CDB, $\overline{EF}$ is parallel to $\overline{DB}$. The length of $\overline{CE}$ is $\frac{1}{3}$ the length of $\overline{CD}$. Label $\overline{ED}$, $\overline{EF}$, $\overline{CF}$, $\overline{FB}$, and $\overline{DB}$ with the correct length for each.

5. Similarly, in figure 2(D), triangle IJH represents half of triangle IGH, with $\overline{IJ}$ a perpendicular bisector of $\overline{GH}$. In triangle IJH, $\overline{KL}$ is parallel to $\overline{JH}$. The length of $\overline{IK}$ is $\frac{1}{3}$ the length of $\overline{IJ}$. Label $\overline{KJ}$, $\overline{KL}$, $\overline{IL}$, $\overline{LH}$ and $\overline{JH}$ with the correct length for each.

6. Compute the lateral surface area of the bull shark tooth, figure 2(A), at $\frac{1}{3}$ the altitude from the tip.

7. Compute the lateral surface area of the blacktip shark tooth, figure 2(B), at $\frac{1}{3}$ the altitude from the tip.

8. Which tooth has the smaller lateral surface area at $\frac{1}{3}$ the altitude from the tip, the bull shark tooth or the blacktip shark tooth?

9. Which tooth is better for puncture?

10. Briefly explain your answer to item 9 above.

We now turn our attention to lateral head-shaking. The shark tooth that will be better at sawing through prey is the one better able to resist the forces applied to the side of the tooth. The ability to resist these forces is determined, in part, by the distance from the side of the tooth to the axis running down the middle of the tooth at a particular altitude. This is represented in the model at $\frac{1}{3}$rd the altitude of the tooth by line segment $\overline{EF}$ for the bull shark tooth, figure 2(C), and line segment $\overline{KL}$ for the blacktip shark tooth, figure 2(D).

Shark Bite Part 2

1. What is the length of $\overline{EF}$?

2. What is the length of $\overline{KL}$?

3. Which tooth is better for sawing through prey during lateral head-shaking?

4. Briefly explain your answer to item 3 above.

This work was supported by a grant from the Florida Department of Education. For additional resources for this lesson visit: http://utweb.ut.edu/rwaggett/science-math-master.html.

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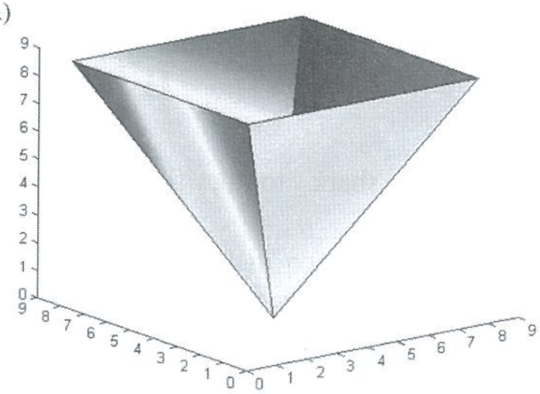
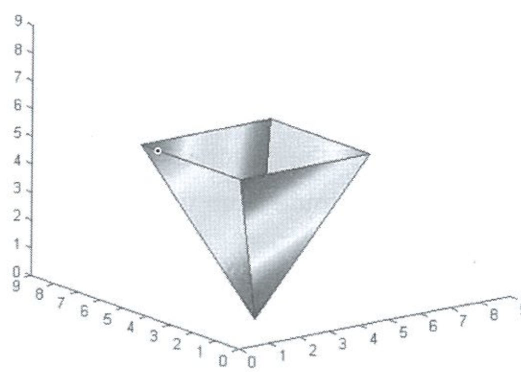
Geometric Analysis of Shark Teeth: Answer Key

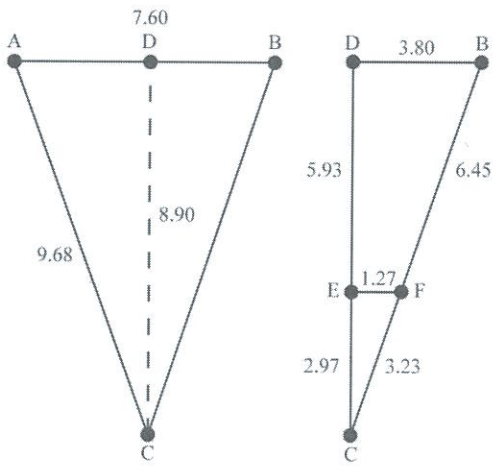
A)



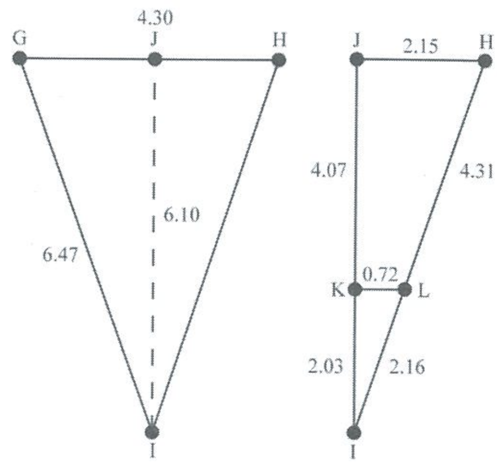
B)



C)



D)



1. If the bull shark and the blacktip shark have the same bite force, will the tooth with the larger or smaller lateral surface area in contact with the prey item have the greater tooth pressure? **smaller**

2. Compute the lateral surface area of the bull shark tooth. **147.14 mm^2**

3. Compute the lateral surface area of the blacktip shark tooth. **55.64 mm^2**

4. In the first model of figure 2(C), which shows triangle CAB, $\overline{CD}$ is a perpendicular bisector of $\overline{AB}$. In the second model, half of triangle CAB is shown. In triangle CDB, $\overline{EF}$ is parallel to $\overline{DB}$. The length of $\overline{CE}$ is $\frac{1}{3}$ the length of $\overline{CD}$. Label $\overline{ED}$, $\overline{EF}$, $\overline{CF}$, $\overline{FB}$ and $\overline{DB}$ with the correct length for each. **See diagram.**

5. Similarly, in figure 2(D), triangle IJH represents half of triangle IGH, with $\overline{IJ}$ a perpendicular bisector of $\overline{GH}$. In triangle IJH, $\overline{KL}$ is parallel to $\overline{JH}$. The length of $\overline{IK}$ is $\frac{1}{3}$ the length of $\overline{IJ}$. Label $\overline{KJ}$, $\overline{KL}$, $\overline{IL}$, $\overline{LH}$ and $\overline{JH}$ with the correct length for each. **See diagram.**

6. Compute the lateral surface area of the bull shark tooth, figure 2(A), at $\frac{1}{3}$ the altitude from the tip. **16.41 mm^2**

7. Compute the lateral surface area of the blacktip shark tooth, figure 2(B), at $\frac{1}{3}$ the altitude from the tip. **6.22 mm^2**

8. Which tooth has the smaller lateral surface area at $\frac{1}{3}$ the altitude from the tip, the bull shark tooth or the blacktip shark tooth? **blacktip shark tooth**

9. Which tooth is better for puncture? **blacktip shark tooth**

10. What is the length of $\overline{EF}$? **1.27 mm**

11. What is the length of $\overline{KL}$? **0.72 mm**

12. Which tooth is better for sawing through prey during lateral head-shaking?
bull shark tooth

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Dr. Rebecca Waggett is an Assistant Professor in the Biology Department at The University of Tampa. Her research areas include the biomechanics, physiology and sensory ecology of zooplankton feeding and escape behavior and the investigation of zooplankton grazer control on the development and maintenance of harmful algal blooms.